

Dynamical analysis of S&P500 momentum

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Abstract

The dynamics of the S&P500 price signal is studied using a moving average technique. Particular attention is paid to intersections of two moving averages with different time horizons. The distributions of the slopes and angle between two moving averages at intersections is analyzed, as well as that of the waiting times between intersections. In addition, the distribution of maxima and minima in the moving average signal is investigated. In all cases, persistent patterns are observed in these probability measures and it is suggested that such variables be considered for better analysis and possible prediction of the trends of the signal.

1 Introduction

Forecasting in empirical finance is based on *recipes* that are often heuristic in nature and specific to the market being considered. Numerous predictive techniques exist,[1] some of which may be theoretically justified to a certain extent, but many others have been proposed in a purely *ad hoc* manner. Technical analysis and the related “charting” methods are therefore often dismissed by academics. Nevertheless, in keeping with a more general trend,[2, 3] it is of interest to see whether statistical physics can bring some insight into the validity or applicability of these recipes. The ultimate and more general goal of such an analysis is to connect optimal strategies to fundamental questions about chaos and deterministic sequences in natural signals.

The moving average (MA) of a stock price is a common tool in technical analysis, frequently used as an indicator for customers to buy or sell stocks.[4, 5] Often, several MAs are studied and their interrelations used as a trading signal. This method may also be applied to indices, sometimes in order to forecast market activities. A MA, $y_T(t)$ at time t of a time-dependent quantity $y(t')$ is defined as:

$$y_T(t) = \frac{1}{T} \sum_{i=t}^{t+T-1} y(i-T) \quad t = T+1, \dots, N, \quad (1)$$

i.e. the average of y over the last T data points prior to time t . Many other forms of MAs exist, such as those involving an exponential smoothing, which tends to dampen out sudden changes. As was shown elsewhere [6] the density of crossing points (ρ) between two MAs is a measure of the signal roughness:

$$\rho \sim \frac{1}{T_2} [(\Delta T)(1 - \Delta T)]^{H-1}, \quad (2)$$

where H is the Hurst exponent and $\Delta T = (T_2 - T_1)/T_2$.

Let us now consider two MAs, y_1 and y_2 , calculated over two different intervals, respectively T_1 and T_2 , with $T_2 > T_1$. [7] In empirical finance the two time intervals often correspond to quite distinct durations, two common choices being a week and a month, and 10 days and 30 days. In those cases it is possible to talk about a long-term and a short-term MA of a given time series. For conciseness we will use the same terminology here (i.e. T_2 is the long-term MA, T_1 is the short-term MA), although in the present analysis occasionally the two time-periods may be quite close. Clearly, if the original signal $y(t)$ increases for a while before decreasing, then y_1 will cross y_2 from above. This event, in which the short-term MA crosses the long-term MA from above, is called a “Death cross” in empirical finance,[8] often interpreted as a “Sell” signal.[4] We will denote it by the symbol D . In contrast, if the short-term MA, y_1 , crosses the long-term MA, y_2 , from below, the crossing point coincides with an upsurge of the signal $y(t)$ and may be taken as a “Buy” signal. This event is called a “Gold cross” and we will denote it by G . Technical financial analysts usually try to “extrapolate” the evolution of y_1 and y_2 , and hence the underlying signal $y(t)$, based on their expectations for the occurrence of G or D crosses. Most computers at brokerages or trading houses are equipped to automatically perform this kind of analysis and trigger the associated activity signals.

As proposed by Vandewalle and Ausloos,[9] one can visualize the change in the *trend* of a signal over some interval T by considering a set of MAs, thus displaying a MA spectrum, taking into account the successive crosses and/or their density. Such a spectrum of MAs is a powerful visualization tool that provides a compact representation of the trends of complex signals.

As noted before, the rate with which MAs vary is of interest to investors who may base their trading strategies on it. It is clear that some uncertainty arises if

G and D crosses are close to each other, or if the angles between crossing MAs are small. Thus, one can argue compellingly that in order to develop a reliable trading strategy, the angle between the MA signals as well as that between the MA signal and the horizontal should also be studied. All of these results will provide information on the slopes of the MA and therefore on the rates of the trend.

The occurrences of G and D crosses are preceded by turnovers in the trend of the MAs. Thus an investigation of the distribution of time intervals between successive minima and maxima is of interest to provide a quantitative measure of the dynamics of the market. In order to gather more information for building an investment strategy based on such a generalized technical analysis, as well as to gain a better understanding of analysis tools for complex time series, we have also studied the distribution of time intervals between successive $[G,G]$, $[D,D]$, $[G,D]$, and $[D,G]$. Finally, we have also analyzed the time interval between successive extrema in the MA as yet another measure of underlying trends.

Our methodology will be generally applicable, but to be specific we will use it here to analyze daily closing prices of the Standard and Poor's 500 index (S&P500) over a 22-year period. We note that Gopikrishnan et al.[10] have studied a related S&P500 time series from a scaling perspective. However, these authors analyzed the signal over different time intervals and with finer time resolution. In addition, they did not address the MA technique.

2 Data and moving average

We consider the S&P500 daily closing price signal from Jan. 01, 1980 to Dec. 31, 2001, as plotted in Fig. 1a, with the associated probability density function (PDF) shown in Fig. 1b. The time series consists of 5556 data points, obtained from Yahoo.[11]. In order to visualize the MAs calculated over different time periods and related crosses between them, Fig. 2 shows two MAs of the S&P500 signal: y_{week} for $T_1 = 1$ week and y_{month} for $T_2 = 1$ month. As is to be expected, the monthly moving average is much smoother than the weekly one.

Following the methodology of Vandewalle and Ausloos,[9] we analyze the density of crossing points between two MAs of the S&P500 closing price signal. This data is plotted in Fig. 3 as a function of $\Delta T = (T_2 - T_1)/T_2$, where T_1 varies between 1 and T_2 and $T_2 = 120$ days. The Hurst exponent estimated from this data, using the relationship given by Eq. (2), is $H = 0.44 \pm 0.01$, in numerical agreement with that obtained by the Detrended Fluctuation Analysis method and clearly distinct from the random walk (RW) value $H = 0.50$. This confirms prior findings[6] that both methods tend to give the same results for H on the order of 0.45.

In order to test the robustness of this result, following the proposal of Viswanathan et al.[12], we shuffled the S&P500 signal in two ways yielding two new (surrogate) time series: one in which the amplitudes were randomly

shuffled, the other where the sign of the S&P500 signal was randomly shuffled. As can be seen in Fig. 1b, the PDF of the fluctuations of the S&P500 closing price signal is characterized by fat tails, which is a well-known result.[10] It is generally accepted[12] that the origin of the fat tailed distributions is a key question to understand financial time series. Most authors believe that the fat tails are caused by long-range volatility correlations. By shuffling the order of the fluctuations the correlations between them are destroyed. The Hurst exponent estimated from the scaling properties of the density of crossing points for the shuffled signal is found to be $H = 0.48 \pm 0.005$ quite close to the RW value indicating a near-absence of long tails. In contrast, destroying only sign correlations, by shuffling the order of the signs (but not the absolute values) of the fluctuations, allows the fat tails to persist: the corresponding roughness exponent is $H = 0.47 \pm 0.01$. In both cases the surrogate data lead to densities of crossing points that scale like a Brownian walk signal (see Fig. 3).

3 Spectrum of moving averages

To visualize the change in the *trend* of the signal over some interval T we consider a set of MAs[9], with the long-term period fixed at $T_2 = 250$ days, i.e. one market year. The short-term period T_1 is varied between 1 and $T_2 - 1$ and the relative difference $\delta = (y_1 - y_2)/y_1$ between the two moving averages is computed.

Fig. 4a represents the resulting spectrum for the S&P500 closing price signal for the period from Jan. 01, 1990 to Dec. 31, 2000. The darker the grey levels the larger the distance (i.e. the absolute value of the difference) between the two MAs.[13] Note the three light grey regions between 1995 and 2000 corresponding to a close proximity ($\delta = 0.05$) between the MAs. This triplet pattern is repeated in rescaled form and for larger separation between the MAs (darker grey levels) in 1997/98 within the middle light grey region and on an even smaller scale further repeated during 1999 within the right-hand-side light grey region. These rescaled patterns correspond to larger differences, $\delta = 0.1$, between the yearly MA and the MAs with $T_1 < 50$ days. In the second half of 1997 one can see the black region corresponding to a large difference, $\delta = 0.15$, between MAs for $T_1 < 40$ days and $T_2 = 250$ days indicating the crash of Oct. 1997. Note that from the point of view of the difference between the MAs, what happened during 1999 looks like a rescaled version of what took place in 1997/98.

For the signals with shuffled order of the fluctuations these characteristic patterns disappeared and are replaced by a rather uniform structure (see Fig. 4b). The apparent streaking in this figure is an artefact due to the cut-off point for the various grey levels, corresponding to $\delta \sim \pm 0$, and does not reflect any underlying periodicity. Thus, we can conclude that the structures seen in Fig. 4a are the result of the long tails in the PDF, which are in turn associated with

volatility correlations. Clearly, then, the MA spectrum contains a great deal of relevant information about the dynamics of the signal. In the next few sections we will see how further dynamic parameters can be extracted from this MA information.

4 Angle distribution at *Gold* and *Death* crosses

In order to estimate the relative position between the two MAs one can measure the angle between them at G and D crosses. The angle between the short- and long-term MAs is a unique measure of the rate of change in the signal and should therefore be able to serve as a useful quantitative indicator of the system dynamics. The angle between two MAs at a G or D cross can appear in three different settings depending on the angles between the MAs and the horizontal (i.e. the slope of the MA at the intersection). In Fig. 5a and 5b these three scenarios are schematically drawn when the intersection is a G , respectively, D cross. Note, for example, that at a G cross the angle at which the short- or long-term MA intersects the horizontal, α , respectively, β , can be positive or negative. However, in all cases the long-term MA crosses the short-term one from above. This mutual relationship is expressed in the distribution of the angle γ between them. Results for the S&P500 closing price signal for the period from Jan. 1, 1980 to Dec. 31, 2001 are shown in Fig. 6(a-f) for G and D crosses.

The distribution of α angles at G crosses for the S&P500 closing price signal for the period from Jan. 1, 1980 to Dec. 31, 2001 is plotted in Fig. 6a as a set of histograms with bin-size 3° . For brevity, in the following, whenever we refer to an angle we will take this as the center of the corresponding bin. Note that there is only one case of a negative angle, the relation schematically drawn in the left-hand-side inset of Fig. 5a. The most frequently observed angle under which the short-term MA intersects the horizontal is 67° . The angles are relatively uniformly distributed between 16° and 85° with additional spikes at 43° , 79° and 85° .

The distribution of the angle β under which the long-term MA intersects the horizontal at a G cross is rather different from that for the angle α , as can be seen in Fig. 6b. This histogram is approximately symmetrical with respect to positive and negative values with maxima at -14° and at 7° . The distribution of the angle γ between the two MAs at a G cross (see Fig. 6c) is rather uniform in the interval $[20^\circ, 50^\circ]$ with a maximum at 26° and a number of low frequency occurrences at 29 , 38 and 46° .

The distribution of angles under which the two MAs intersect the horizontal and with one another at D cross is plotted in Fig. 6(d-f). The angle α of the short-term MA is mostly negative with one case of positive 60° which corresponds to the case sketched in the right-hand-side inset of Fig. 5b. At the time of crossings the α angles of the short-term MA are paired with β angles whose

distribution is plotted in Fig. 6e. Therefore, the negative α angles and positive β angles correspond to the schematic representation in Fig. 5b, while negative α and negative β values represent the case sketched in the left-hand-side inset in Fig. 5b. The maximum value of the α angle is at -82° . One is very unlikely to observe an angle of -47° in the data. The maximum β value is equal to 7° , so the long-term MA is most likely to be rather close to the horizontal with a quasi-homogeneous spread over the interval $[-10^\circ, 10^\circ]$. In comparison the spread of β values at G crosses reported above is a bit wider. The most likely angle γ between the MAs at D cross can have two values 23° and 59° with 33% less chance to have a value of 47° .

Clearly, the distribution of the slopes of the MAs at crosses as well as the angles between them is very rich and far from random. It remains to be seen if similar patterns can be detected in other financial time series and if their occurrence can be related to features of the spectrum. However, it seems evident that more sophisticated prediction strategies should take these variables into account.

Next, we have studied the distribution of the time intervals between successive $[G,G]$, $[D,D]$, $[G,D]$, and $[D,G]$ crosses for the S&P500 closing price signal in two distinct time intervals chosen because they differ strongly in their investment environment. These are the time periods from Jan 1, 1980 to Dec 31, 1990 (see Fig. 7(a-d)) and from Jan 1, 1991 to Dec 31, 2001 (see Fig. 7(e-h)), the former generally characterized by a rather “sluggish” economy, the latter mainly reflecting the “bull market” of the 1990s.

While the shortest interval between successive $[G,G]$ crosses in the first case appears to be 3 days, during the second period it is twice as long, i.e. 6 days. The maximum of the distribution is observed to occur at 9 and 13 for the first period and at 17 days for the second period, and is thus not drastically different between the two periods. The most probable time interval between successive $[D,D]$ crosses during the first period is 19 days. Time intervals of similar length: 20-22 days, e.g. approximately one market month, are very unlikely to occur during the second period, whose maxima are at 16 and 28 days. Thus, downturns tended to occur more frequently in the 1980s than in the 1990s. A measure of the time necessary for the market to recover is the interval between $[G,D]$ crosses. Its maximum is at 4 days during the first period. In sharp contrast, the second period is characterized by a high frequency of occurrence of $[G,D]$ crosses for time intervals between 2 to 6 days, indicating that the market tended to recover faster during the second observation interval.

The way the market is going down from a D to G cross is reflected in the distribution of time intervals between consecutive D and G crosses. During both periods, the most likely interval between successive D and G crosses is 4 days.

5 Minima-Maxima distribution

As can be noticed in Fig. 2, the occurrences of G and D crosses are preceded by turnovers of the trend of the MAs. Thus the distribution of time intervals between successive minima and maxima can be calculated in order to provide information on the dynamics of the market. To this end for the S&P500 signal studied here the PDFs of the time interval between successive maxima $[M_1, M_2]$, minima $[m_1, m_2]$, maxima-minima $[M_1, m_1]$, and minima-maxima $[m_1, M_1]$, are plotted in Fig. 8(a-h) for the short-term, y_{week} , and long-term, y_{month} , MAs. The x-axis of all data in Fig. 7(a-h) is chosen in such a way that the bin of each x-axis is equal to one day. Also the ticks are associated with the middle of the bin, i.e. to 0.5. For the purpose of comparison the insets of Fig. 8(c-h) have their x- and y-axis limits the same as those of Fig. 8(a,b). It is of interest to note that the PDFs of $[M_1, M_2]$ and $[m_1, m_2]$ for the short-term MA, y_{week} , are very similar, which is not surprising, both with a maximum at 2 days. In contrast, the $[m_1, m_2]$ PDF for the long-term MA, y_{month} , exhibits a maximum at 2 days and a rather homogeneous distribution for time intervals between 3 and 11 days, while the respective maxima-minima $[M_1, m_1]$ PDF has the appearance of an exponential one, although the scarcity of data does not allow us to make this statement rigorous.

We also checked the frequency of appearance of combinations of positive and negative increments of the MAs. Let a positive increment be marked by 1 and a negative increment be marked by -1. Then we test the frequency of occurrence of time intervals between two consecutive triplets (1 1 -1) and between two consecutive triplets (-1-1 1) for the two MAs of interest. These data are shown in Fig. 9 for the weekly and monthly MA and the entire 22-year period. The increment of the histogram is 1 day. The weekly MA y_{week} has a maximum of [(1 1 -1),(1 1 -1)] time interval at 9 days. The maximum of the [(-1-1 1),(-1-1 1)] interval for y_{week} is at 8 days with very low probability of occurrence at 4 days time interval. The maxima for [(1 1 -1),(1 1 -1)] is at 5 days and for [(-1-1 1),(-1-1 1)] at 3 days for the monthly MA y_{month} , while it is very unlikely that (1 1 -1) will be followed by (1 1 -1) after 21 days. There are also many time intervals that have zero probability to occur for [(-1-1 1),(-1-1 1)]. They are 20, 21, 30, 36, 39, 48, 50, 51, 53, 56, 61 days, to mention some of them.

6 Conclusion

We presented a set of quantitative measures of two MAs of the S&P500 daily closing price signal and of their relative position to one another. Since different MAs represent the trend of the signal looked upon from different investment horizons, the measures of their relative geometry provide information on the change of the trend of the signal. The distribution of the time interval between DD , GG , DG and GD crosses, together with that of the maxima and minima in

the MA, and the distribution of the angle between the two MAs at the crosses can serve as ingredients for an investment strategy that enriches the classical technical analysis with parameters of the dynamics of the signal trend, i.e. of the momentum of S&P500 closing price. We emphasize, that these conclusions should not be taken as an endorsement of chartist strategies or technical analysis in general. Whether it is possible to base a successful trading strategy on the rather subtle patterns observed here remains to be seen and has not been addressed in this paper.

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- [7] Strictly speaking, in accord with the notation established in Eqn. (1), these should be labelled y_{T_1} and y_{T_2} , but we will prefer the lighter notation established in the main text whenever no confusion can arise.
- [8] The vivid terms “death” cross and “gold” cross were introduced in the physics literature by Vandewalle and Ausloos.[9] They are commonly used in the French financial literature (“croix de mort” and “croix dorée”), although they are not as widespread in anglophone financial sources.
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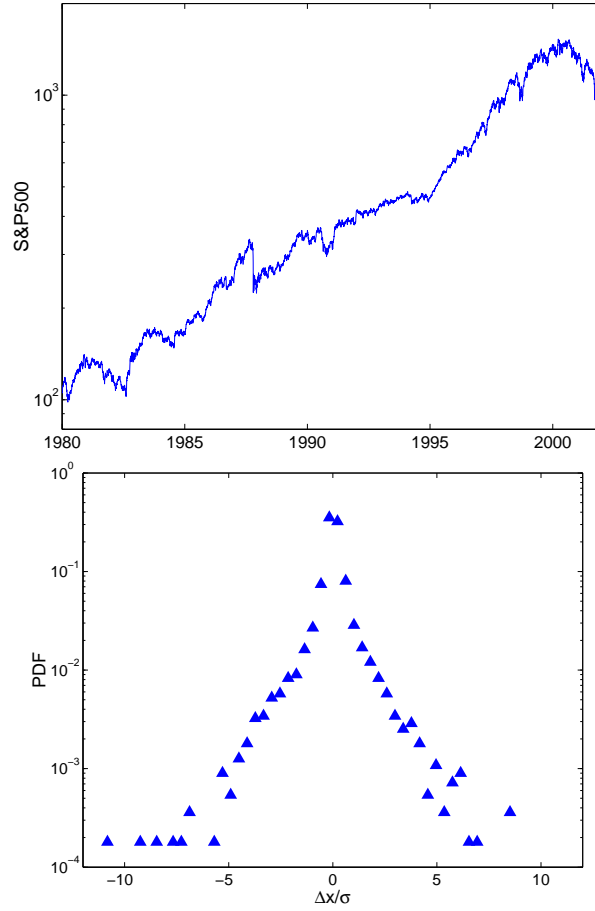


Figure 1: (a) S&P500 daily closing value signal between Jan. 01, 1980 and Dec. 31, 2001, i.e. 5556 data points. (b) Probability density function of fluctuations of S&P500 signal.[11]

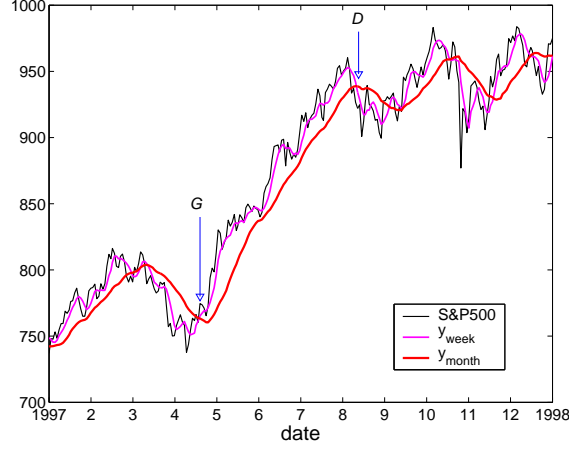


Figure 2: Typical S&P500 daily closing value signal between Jan. 01, 1997 and Dec. 31, 1998, with two moving averages, y_{week} and y_{month} for $T_1 = 5$ days and $T_2 = 21$ days. D and G crosses, are defined in the text.

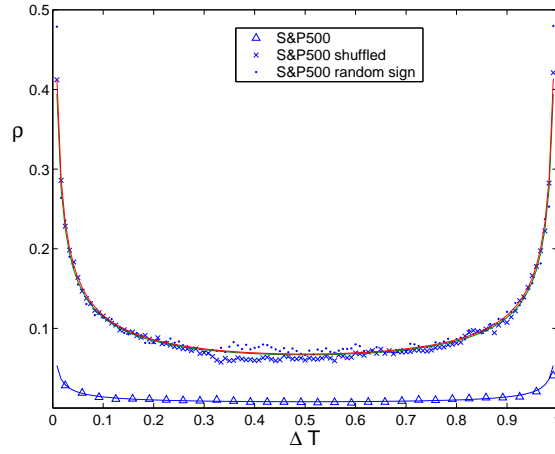


Figure 3: The density ρ of crossing points as a function of the relative difference ΔT with $T_2 = 120$ days. S&P500 daily closing value signal (triangles), S&P500 shuffled values signal (x), and S&P500 with randomized sign and preserving the amplitude of the signal (dots). Continuous lines are fits to Eq. (2).

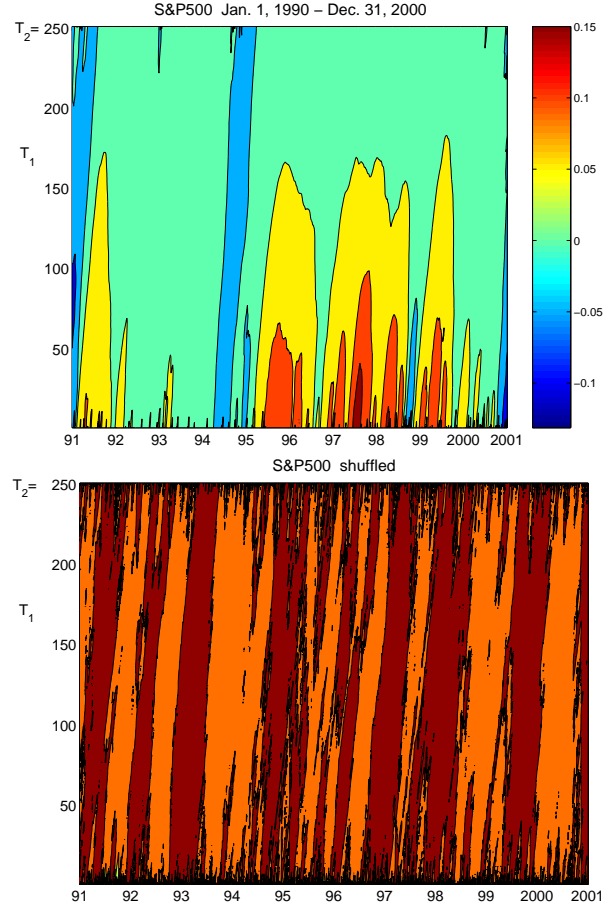


Figure 4: (a) The spectrum of moving averages for the S&P500 daily closing value signal data between Jan. 01, 1990 and Dec. 31, 2000. The y-axis corresponds to T_1 . The long term is fixed to $T_2 = 252$, i.e. one market year. (b) The spectrum of moving averages for the S&P500 shuffled signal. The grey levels correspond to the relative distance δ between the two moving averages. The darker the grey level the larger the difference.

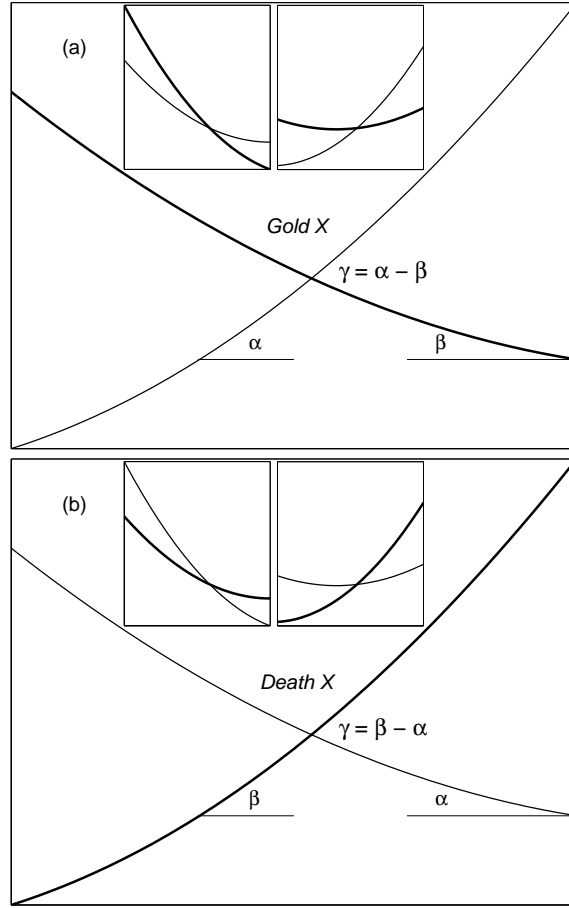


Figure 5: (a) Schematic presentation of a *Gold cross* between two moving averages. (b) Schematic presentation of a *Death cross* between two moving averages. The thicker lines correspond to the longer-term moving average. Noted are the angles between the horizontal and the short- (α) and long-term (β) moving averages; γ denotes the angle between both moving averages.

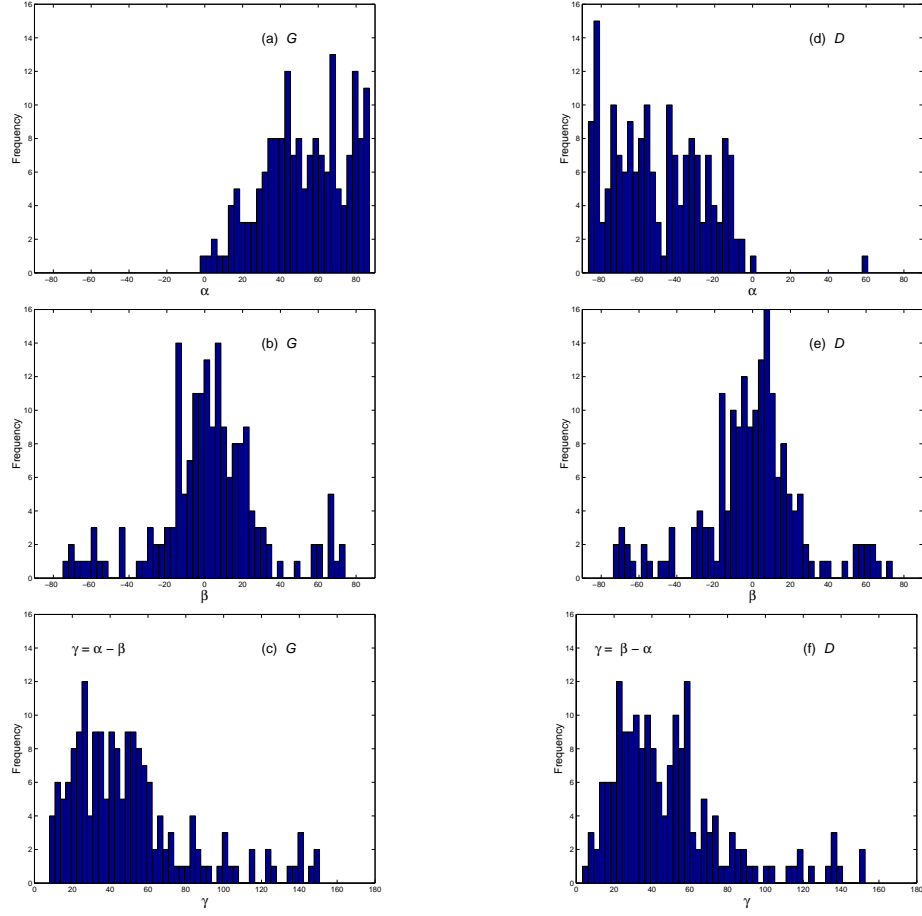


Figure 6: Frequency of the angle between the horizontal and the short-term moving average α : (a) for G crosses and (d) for D crosses; between the horizontal and the long-term moving average β : (b) for G crosses and (e) for D crosses; between the two moving averages γ : (c) for G crosses and (f) for D crosses. Data considered are the S&P500 closing price from Jan 1, 1980 to Dec 31, 2001.

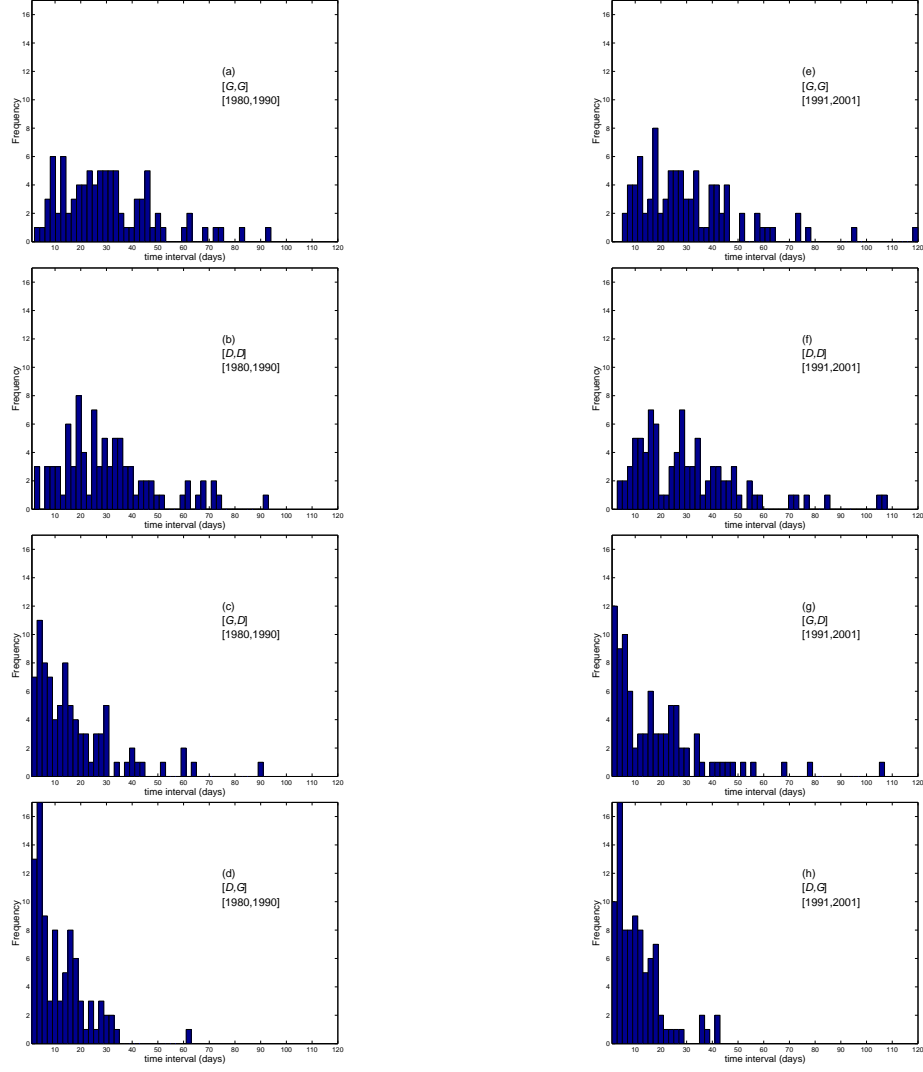


Figure 7: Frequency of time interval between consecutive D and G crosses in different combinations, e.g. GG, DD, GD, DG for the S&P500 closing price signal. Two periods are considered: (a-d) from Jan 1, 1980 to Dec 31, 1990; (e-h) from Jan 1, 1991 to Dec 31, 2001.

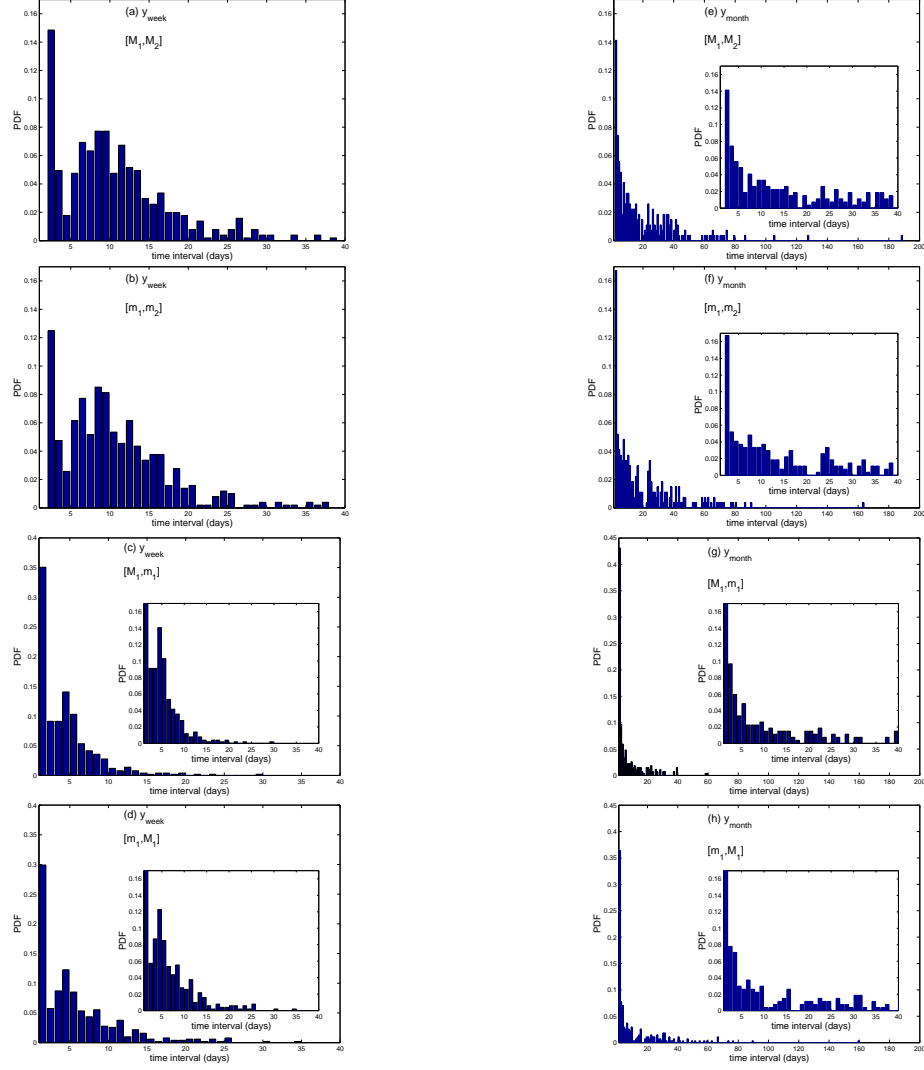


Figure 8: Probability density function of time interval between consecutive (a) *maxima* and (b) *minima* for 5-day (weekly) moving average y_{week} , and (c) *maxima* and (d) *minima* for 21-day (monthly) moving average y_{month} of the S&P500 closing price signal for the period from Jan 1, 1980 to Dec 31, 2001.

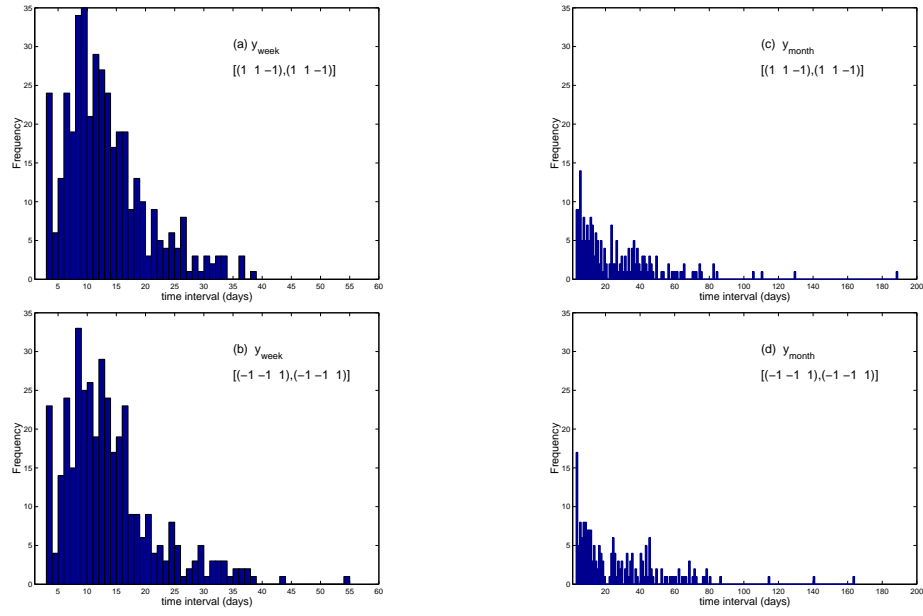


Figure 9: Frequency of time interval appearance between consecutive (a) (1 1 -1) and (b) (-1 -1 1) for 5-day (weekly) moving average y_{week} , and (c) (1 1 -1) and (d) (-1 -1 1) for 21-day (monthly) moving average y_{month} of the S&P500 closing price signal for the period from Jan 1, 1980 to Dec 31, 2001.